Pergamon

www.elsevier.com/locate/jappmathmech

PII: S0021-8928(00)00097-6

J. Appl. Maths Mechs, Vol. 64, No. 5, pp. 691–700, 2000

0021-8928/00/\$—see front matter

THE EFFECT OF THE ATMOSPHERE ON THE ATTITUDE MOTION OF A DUMB-BELL-SHAPED ARTIFICIAL SATELLITE†

V. V. BELETSKII and M. L. PIVOVAROV

Moscow

e-mail: beletsky@spp.keldysh.ru; mp@mx.iki rssi.ru

(*Received* 29 *May* 2000)

An orbital two-body system – two point masses connected by an ideally flexible massless inextensible thread – is considered. Equations are obtained for the plane relative motion of the system on the taut thread and a condition for such motion to exist is derived, on the assumption that the centre of mass of the system describes a Keplerian elliptic orbit. The torques of gravitational forces, aerodynamic pressure, aerodynamic friction and the aerogradient effect are taken into account. The aerogradient effect may lead to strong spinup of the satellite, while the orbit ellipticity may cause chaotization of the motion.

1. THE EQUATIONS OF MOTION AND THE CONNECTEDNESS CONDITION

Consider an orbital two-body system – two point masses m_1 and m_2 connected by an ideally flexible massless inextensible thread, acted upon by gravitational and aerodynamic forces applied to the points m_1 and m_2. It is assumed that the centre of mass of the system moves along a Keplerian elliptic orbit. It has been shown [1]‡ that suitable control forces applied to the points m_1 and m_2 will fully compensate for the deviations of the orbit of the centre of mass from a Keplerian orbit, without at the same time generating torques in the relative motion of the system. If such control forces are not applied, it is assumed that the deviation of the orbit from a Keplerian one may be neglected.

We will introduce the following notation (Fig. 1): O is the centre of attraction (i.e. the Earth's centre), C is the centre of mass of the dumb-bell, which is describing a Keplerian elliptic orbit, $Cr\tau$ is an orbital frame of reference, where $\mathbf{r}$ is the unit vector in the direction of the instantaneous radius vector of the orbit, τ is the unit vector in the transverse direction, pointing in the direction of the orbital motion, $\mathbf{R}$ is the radius vector of the dumb-bell's centre of mass, $|\mathbf{R}| = R$ is the instantaneous distance from the attracting centre O to the dumb-bell's centre of mass C, $R\pi$ is the minimum (perigee) value of that distance, ν is the true anomaly (the angle between the perigee and the instantaneous radius vectors of the centre of mass C), $\mathbf{V}$ is the instantaneous velocity vector of the centre of mass C, m_i are point masses at the ends of the dumb-bell, $\mathbf{r}_i$ are vectors from the centre of mass C to the ends of the dumb-bell, $l_i = |\mathbf{r}_i| = l(m - m_i)/m$, $m = m_1 + m_2$, $l = l_1 + l_2$ is the length of the dumb-bell, $|\mathbf{R}_i| = R_i$ is the distance from the attracting centre O to the mass m_i, $\mathbf{R}_i = \mathbf{R} + \mathbf{r}_i$, Ω is the absolute angular velocity of the dumb-bell, $\nu = d\nu/dt$ is the instantaneous angular velocity of rotation of the dumb-bell's centre of mass, $\alpha = d\alpha/dt$, where α is the angle between the transversal τ and the direction of $\mathbf{r}_2$, measured in the direction of orbital motion and $\mathbf{v}_i = \Omega \times \mathbf{r}_i$, $\mathbf{V}_i = \mathbf{V} + \mathbf{v}_i$; throughout this paper, $i = 1, 2$.

We know from the theory of Keplerian orbits that

$$V_r = \sqrt{\frac{\mu}{p}} e \sin\nu, \quad V_\tau = \sqrt{\frac{\mu}{p}}(1 + e\cos\nu), \quad V = \sqrt{\frac{\mu}{p}}\Delta \tag{1.1}$$

$$\Delta = \sqrt{1 + e^2 + 2e\cos\nu}, \quad R = \frac{p}{1 + e\cos\nu}, \quad p = R_\pi(1 + e), \quad \dot{\nu} = \frac{d\nu}{dt} = \frac{\sqrt{\mu p}}{R^2}$$

†*Prikl. Mat. Mekh.* Vol. 64, No. 5, pp. 721–731, 2000.

‡See also: BELETSKII, V. V., VORONTSOVA, V. A., KOF, L. M. and PANKOVA, D. V., The effect of aerodynamics on the relative motion of an orbital two-body system, Part I. Regular motions Preprint No. 38. M. V. Keldysh Inst. Prikl. Mat., Ross. Akad. Nauk. Moscow, 1996.

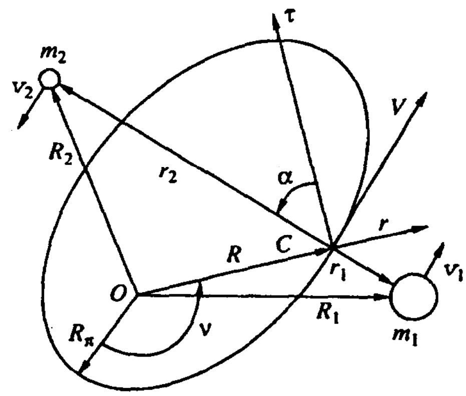

Fig. 1

where e is the eccentricity, p is the focal parameter of the orbit, μ is the gravitational constant, V_r and V_τ are the radial and transversal components of the velocity of the centre of mass C, and V is the magnitude of the velocity. We take the aerodynamic force acting on the point of mass m_i to be

$$\mathbf{F}_i = -C_i\sigma(R_i)V_i\mathbf{V}_i, \quad V_i = |\mathbf{V}_i|$$

The components of the vectors $\mathbf{V}_i$ relative to the r and τ axes are given by the formulae

$$V_{ir} = \sqrt{\frac{\mu}{p}}\, e\sin\nu - (-1)^i l_i\Omega\cos\alpha, \quad V_{i\tau} = \sqrt{\frac{\mu}{p}}(1 + e\cos\nu) - (-1)^i l_i\Omega\sin\alpha$$

C_i are coefficients having the dimension of area, which are constant in this formulation of the problem (it may be assumed, for example, that $C_i = \pi\rho_i^2$, if one is considering a system of two spheres of radii ρ_i and the interaction of the molecules of the flow with the spheres is absolutely inelastic [2]) and $\sigma(R_i)$ is the density of the atmosphere at a distance R_i from the attracting centre, taken to be

$$\sigma(R_i) = \rho_\pi \exp\left(-\frac{R_i - R_\pi}{H}\right), \quad R_i = [R^2 + l_i^2 - (-1)^i 2Rl_i\sin\alpha]^{1/2}$$

where H is the so-called atmosphere scale height, which is taken to be constant. The parameter ρ_π has the meaning of the density of the atmosphere at the perigee of the orbit. We will also need the value of the density of the atmosphere at the dumb-bell's centre of mass C, that is, at a distance R from the attracting centre. Using relations (1.1), we obtain

$$\sigma(R) = \rho_\pi \exp\left(-\frac{R - R_\pi}{H}\right) = \rho_\pi\sigma_*(\nu),$$

$$\sigma_*(\nu) = \exp\left(-\varkappa\frac{1-\cos\nu}{1+\cos\nu}\right), \quad \varkappa = \frac{R_\pi}{H}e$$

The gravitational forces have the form

$$\mathbf{G}_i = -\frac{\mu m_i\mathbf{R}_i}{R_i}$$

It is natural to assume that

$$l_i \ll R, \quad l_i\Omega \ll V$$

that is, the dumb-bell is sufficiently small compared with the orbital radius, and the linear velocity of rotation of the dumb-bell is small compared with the orbital velocity. It will therefore suffice to consider the acting forces accurate up to the leading terms of their expansions in terms of the parameters l_i. As we know, in the expansion of expressions for gravitational forces it is sufficient to retain terms of order up to and including two [3]. For aerodynamic forces it is sufficient to use expansions up to first-order terms inclusive.

Now, considering relative motion in the orbital reference frame $Cr\tau$ and utilizing the Lagrange equation of the first kind [3], we obtain equations of coupled motion for the dumb-bell and conditions for the connection to be maintained

$$\frac{d^2\alpha}{dv^2} - 2\rho e \sin v\left(\frac{d\alpha}{dv}+1\right) - 3\rho\sin\alpha\cos\alpha - a\rho^4\sigma_*\Delta^2\sin\delta - $$
$$-k\rho^4\sigma_*\Delta^2\sin\delta\sin\alpha + b\rho^2\sigma_*\Delta(1+\sin^2\delta)\left(\frac{d\alpha}{dv}+1\right) = 0 \tag{1.2}$$

$$\left(\frac{d\alpha}{dv}+1\right)^2 + \rho(3\sin^2\alpha - 1) - a\rho^4\sigma_*\Delta^2\cos\delta - k\rho^4\sigma_*\Delta^2\cos\delta\sin\alpha +$$
$$+ b\rho^2\sigma_*\Delta\left(\frac{d\alpha}{dv}+1\right)\sin\delta\cos\delta > 0 \tag{1.3}$$

where

$$\rho = \frac{1}{1+e\cos v}$$
$$\sin\delta = \frac{1}{\Delta}[(1+e\cos v)\sin\alpha + e\sin v\cos\alpha]$$
$$\cos\delta = \frac{1}{\Delta}[(1+e\cos v)\cos\alpha - e\sin v\sin\alpha]$$

The constant dimensionless parameters have the following meanings

$$a = \rho_\pi p^2 \frac{C_2 m_1 - C_1 m_2}{l m_1 m_2}, \quad k = \frac{bp}{H}, \quad b = \rho_\pi P \frac{C_2 m_1^2 + C_1 m_2^2}{m_1 m_2 (m_1 + m_2)} \tag{1.4}$$

The focal parameter p is related to the eccentricity e of the orbit and to the magnitude R_π of the radius vector of the perigee of the orbit by formula (1.1). Explicit formulae for $\sigma_*(\upsilon)$ and $\Delta(\upsilon)$ were presented above.

2. DISCUSSION OF THE EQUATION OF MOTION AND THE CONNECTEDNESS CONDITION

Research into the influence of aerodynamic effects on the spin and attitude of artificial satellites began in the 1950s and are still going on. This research is reflected, for example, in the monographs [2–5] and in hundreds of journal publications. New interest in this field arose in relation to projects of large-scale tethered systems [1], where the aerogradient effect, for example, is significant [2], as is the possibility of chaotization of the motion due to impact arrivals at the connection [6].† Embedded in Eq. (1.2) is a different mechanism of possible chaotization – due to the motion of the system in an elliptic orbit, which causes the non-linear equation (1.2) to be non-autonomous.

Equation (1.2) and the connectedness condition (1.3) for the motion allow for the ellipticity of the orbit (the eccentricity of the orbit $e \neq 0$), the gravity-gradient effect for the dumb-bell (the third term in Eq. (1.2)), aerodynamic pressure effects ($a \neq 0$), aerodynamic friction ($b \neq 0$) and the aerogradient effect ($k \neq 0$). Note that if the masses m_1 and m_2 are connected by an absolutely rigid rod (a rigid dumb-bell), the connectedness condition is not taken into consideration, as the motion is always connected, by definition, and is described by Eq. (1.2) alone.

†See also BELETSKII, V. V. and PANKOVA, D. V. The effect of aerodynamics on the relative motion of an orbital two-body system, Part 2. Chaotic and regular motions. Preprint No. 40. M. V. Keldysh, Inst. Prikl. Mat, Ross. Akad. Nauk, Moscow, 1996.

Equation (1.2) and condition (1.3) in this general form are apparently being published here for the first time. Various special cases of (1.2) and (1.3) have been published and investigated.

We present a brief survey of the relevant publications.

Circular orbit: $e = 0$. Equation (1.2) was considered in [4] only taking into account the effect of the torque due to aerodynamic pressure ($a \neq 0$) together with the torque due to the gravitational gradient, for which the following general formula for the gravity-gradient coefficient was assumed

$$n^2 = 3(A-C)/B \tag{2.1}$$

where A, B and C are the principal central moments of inertia of the satellite, considered as an arbitrary rigid body. In the case of a dumb-bell satellite, $C = 0$, $A = B$ and $n^2 = 3$, as indeed assumed in Eq. (1.2).

The problem of the motion of an orbital two-body system was first considered in [7], where only the gravitational torque was taken into account, ignoring aerodynamic effects. In other words, Eq. (1.2) and condition (1.3) are derived and investigated in [7] for $e = 0$, $a = b = k = 0$. The problem of the dynamics of orbital two-body systems as systems with a unilateral constraint was first formulated and investigated in [7, 8]. These investigations are also reflected in the monographs [1, 3]. The effect of aerodynamic pressure in this problem was first considered in [9]. The equation of motion and connectedness condition obtained in [9] follow from (1.2) and (1.3) with $e = 0$, $b = k = 0$, $a \neq 0$ (see also the first footnote).

The complete equation of motion and the connectedness condition for a circular orbit ($e = 0$, $a \neq 0$, $k \neq 0$, $b \neq 0$) were first derived in the study cited in the first footnote and, in an improved version, in [10]. This equation and condition are identical with (1.2) and (1.3) when $e = 0$.

Elliptic orbit. For an elliptic orbit ($e \neq 0$) in the purely gravitational case (all the aerodynamic parameters (a, b, k) in Eq. (1.2) vanish), Eq. (1.2) was first published for the general gravitational parameter (2.1) in [11]; since then several dozen studies have been devoted to investigations of the equation.

Equation (1.2) for the motion of a dumb-bell in the purely gravitational case was first accompanied in [12] by the condition (1.3) of coupled motion for that case ($a = b = k = 0$). The condition from [12], of course, considered for a circular orbit ($e = 0$), is identical with the connectedness condition obtained in [7]. Equation (1.2) has been considered, ignoring the dissipative and aerogradient terms ignored ($b = k = 0$), but with the general form (2.1) of the gravitational parameter and an arbitrary value of the aerodynamic pressure parameter ($a \neq 0$); the equation has been analysed for the existence and stability of three-parameter (e, n^2, a) 2π-periodic motions of a satellite.†‡

3. ANALYSIS OF THE PARAMETERS OF THE EQUATION OF MOTION

The qualitative and quantitative characteristics of the solutions of Eq. (1.2) depend on the absolute values of the dimensionless parameters of the aerodynamic pressure a, the aerodynamic gradient k, the aerodynamic friction b and also on the value of the quotients of these parameters to each other and to the number 3, which characterizes the effect of the gravitational gradient on a dumb-bell satellite.

The effect of the ellipticity of the orbit is determined by the value of its eccentricity e and very significantly by the value of the parameter $\varkappa = R_\pi e/H$ which occurs in the aerodynamic terms of Eq. (1.2). The parameter $\varkappa$ is two orders of magnitude greater than the orbit eccentricity e. This means that even at very small orbit eccentricities the aerodynamic terms may exert *a* significant influence on the motion, including its chaotization.

Let us estimate the relative influence of the aerodynamic and gravitational parameters. It is clear from relations (1.4) and (1.1) that the ratio

$$\frac{b}{k} = \frac{H}{R_\pi(1+e)} \tag{3.1}$$

†MEL'NIK, N. V. Periodic oscillations of an artificial satellite in a circular orbit taking the effect of atmospheric drag into account. Preprint No. 97. M. V. Keldysh, Inst. Prikl. Mat., Ross. Akad. Nauk, Moscow, 1976.

‡MEL'NIK, N. V. 2π-periodic oscillations of an artificial satellite in the plane of an elliptical orbit in the presence of atmospheric drag. Preprint No. 119. M. V. Keldysh, Inst. Prikl. Mat., Ross. Akad. Nauk, Moscow, 1976.

depends neither on the size of the dumb-bell satellite nor on its dynamic characteristics, but only on the parameters of the orbit and the atmosphere scale height H. Since $R_\pi \sim 6000$ km and $H \sim 30$–60 km, it follows that $b/k \sim 0.005$–0.01, that is, the aerodynamic dissipation coefficient is two orders of magnitude less than the aerogradient coefficient. Thus, the aerogradient effect is more significant than the effect of aerodynamic friction. Of course, under conditions of free-molecular flow, the concept of aerogradient is rather artificial for a satellite of small size. But for orbital tethered systems with a fairly long tether, the aerodynamic gradient (the density gradient of the atmosphere) is a wholly real phenomenon and, as is obvious from (3.1), it exceeds the aerodynamic friction effect by two orders of magnitude.

If $C_2/m_2 \gg C_1/m_1$, the relative influence of the aerodynamic factors is estimated by the relations

$$\frac{k}{a} \sim \frac{l}{H}, \quad \frac{b}{a} \sim \frac{l}{R_\pi(1+e)}$$

and, depending on the tether length, we have the following numerical values

l, km	0.06	0.6	6	60
k/a	0.001	0.01	0.1	1
b/a	10^{-5}	10^{-4}	10^{-3}	10^{-2}

It is assumed (without loss of generality) that in (1.2)–(1.4) we always have $a \geqslant 0$, that is

$$\frac{C_2}{m_2} - \frac{C_1}{m_1} \geqslant 0 \qquad (3.2)$$

This corresponds to a simply defined enumeration of the points m_i. Namely: the index $i = 2$ is assigned to the point whose characteristics m_2, C_2 satisfy condition (3.2). By virtue of this condition, the parameter a may take values from zero (when the numbers C_2/m_2 and C_1/m_1 are identical, which happens, e.g. in a dumb-bell with parameters $m_1 = m_2$ and $C_1 = C_2$) to very large numbers (when m_2 and C_2 correspond to an inflated balloon – an aerodynamic stabilizer of small mass and large size).

Note that if the altitude of the centre of mass of the system above the Earth's surface is $h \sim 200$–225 km, then $\rho_\pi R_\pi^2 \sim 3 \times 10^{-2}$ g/cm^2, so that values $k \sim a \sim 1$ are attained when $C_2/m_2 \sim 30$ cm^2/g (an inflated balloon of large size $P \sim H \sim 60$ km). For a satellite-probe with $C_2/m_2 \sim 10^{-2}$ cm^2/g and tether length ~ 1 km, estimates give $a \sim 10^{-2}$, $k \sim 10^{-3}$ and the relative motion of the system is determined mainly by the gravity-gradient torque, while aerodynamic effects play the role of a small perturbing factor.

4. DYNAMICS OF A DUMB-BELL SATELLITE IN A CIRCULAR ORBIT

In a circular orbit, $e = 0$ and relations (1.2) and (1.3) become [10]

$$\frac{d^2\alpha}{d\nu^2} - 3\sin\alpha\cos\alpha - a\sin\alpha - k\sin\alpha^2 + b(1+\sin^2\alpha)\left(\frac{d\alpha}{d\nu}+1\right) = 0 \qquad (4.1)$$

$$\left(\frac{d\alpha}{d\nu}+1\right)^2 + 3\sin^2\alpha - 1 - a\cos\alpha - k\sin\alpha\cos\alpha + b\left(\frac{d\alpha}{d\nu}+1\right)\sin\alpha\cos\alpha > 0 \qquad (4.2)$$

When there is no dissipation ($b = 0$), Eq. (4.1) has a first integral

$$\frac{1}{2}\left(\frac{d\alpha}{d\nu}\right)^2 - \frac{3}{2}\sin\alpha^2 + a\cos\alpha - \frac{k}{2}\left(\alpha - \frac{1}{2}\sin 2\alpha\right) = h \qquad (4.3)$$

which determines the phase portrait of the problem (h is an integration constant), from which, using condition (4.2) with the inequality sign reversed, the zones of leaving the constraint are eliminated.

We will describe the basic properties of relations (4.1)–(4.3), following results previously obtained [10] (which were reproduced in [13]).

The stationary points of Eq. (4.1) are determined by the solutions in $[0, 2\pi)$ of the equation

$$\sin\alpha(3\cos\alpha + k\sin\alpha + a) = 0 \qquad (4.4)$$

This equation generally has four roots in the half-closed interval $[0, 2\pi)$

$$\alpha_1 = 0, \quad \alpha_2 = \pi, \quad \alpha_3 = \alpha_+, \quad \alpha_4 = \alpha_- \tag{4.5}$$

where the roots α_+ and α_- are defined by

$$\cos\alpha_\pm = \frac{-3a \pm kw}{9+k^2}, \quad w = \sqrt{9+k^2-a^2} \tag{4.6}$$

Hence it follows that if

$$a^2 \geqslant 9 + k^2 \tag{4.7}$$

then Eq. (4.1) has only two stationary points. If the inequality opposite to (4.7) holds, there are four stationary points. The case $w = 0$ corresponds to the presence of multiple roots.

The stability (instability) of the stationary points α_j ($j = 1, 2, 3, 4$) depends on the sign of the coefficient A_j in the equation in variation $\delta\alpha$ obtained by varying the initial equation (4.1) about the stationary points

$$\frac{d^2(\delta\alpha)}{dv^2} + A_j\delta\alpha = 0, \quad j = 1,2,3,4$$

The index of the coefficient A_j corresponds to the index of the solution α_j of Eq. (4.4). It turned out that

$$A_1 = -(3 + a), \quad A_2 = a - 3$$

$$A_3 = \frac{w}{9+k^2}(ka+3w), \quad A_4 = \frac{w}{ka+3w}(9-a^2) \tag{4.8}$$

Positive values of A_j correspond to a stable stationary point (a centre) and negative values to an unstable point (a saddle).

It follows from (4.5), (4.6) and (4.8) that the following possibilities exist (Fig. 2).

1. $a^2 > 9 + k^2$. There are only two stationary points: $\alpha_1 = 0 - a$ saddle, and $\alpha_2 = \pi - a$ centre. In physical terms, this corresponds to the strong action of an aerodynamic stabilizer and a stable position of the dumb-bell along the tangent to the orbit (the stabilizer is behind the dumb-bell's centre of mass).

2. $9 < a^2 < 9 + k^2$. There are four stationary points $\alpha_1 = 0 - a$ saddle, $\alpha_2 = \pi - a$ saddle, $\alpha_3 = \alpha_+ - a$ centre and $\alpha_4 = \alpha_- - a$ centre.

The separatrices emerge from the saddle and are constructed according to integral (4.3). In case 1 the infinite "whisker" of the separatrix winds spirally around the phase cylinder in the half-space $d\alpha/dv < 0$, enters the point $\alpha = 0$, $d\alpha/dv = 0$, and then forms a closed loop around the centre $\alpha = \pi$, $d\alpha/dv = 0$, enters the saddle from the half-space $d\alpha/dv > 0$ and emerges from the saddle, winding around the phase cylinder in an infinite spiral "whisker" in the half-space $d\alpha/dv > 0$. At the same time, $d\alpha/dv \to +\infty$.

All the phase trajectories inside the closed loop of the separatrix are closed and define periodic oscillations about a stable relative equilibrium point. The presence of the infinite "whiskers" of the separatrix means that, on all other trajectories (not situated within the separatrix loop), the angular velocity increases without limit (the aerodynamic spinup effect). We recall that aerodynamic friction is being neglected in this treatment. When friction is present, the phase trajectories either contract to a relative equilibrium (somewhat displaced because of the dissipation coefficient) or tend to a limit cycle of the second kind [14] surrounding the phase cylinder. The mean value of the dimensionless absolute angular velocity ω of the dumb-bell in the cycle is given approximately by the formula

$$\omega \approx \frac{1}{3}\frac{k}{b} = \frac{1}{3}\frac{R_c}{H}$$

where R_c is the radius of the circular orbit of the mass centre of the dumb-bell and H is the atmosphere scale height. Estimates give $\omega = 30$, that is, the dumb-bell rotates 30 times more rapidly than it moves in its orbit. This happens when the dimensional value of the angular velocity of rotation of the dumb-bell is $\omega_p \sim 20$, which is large but finite.

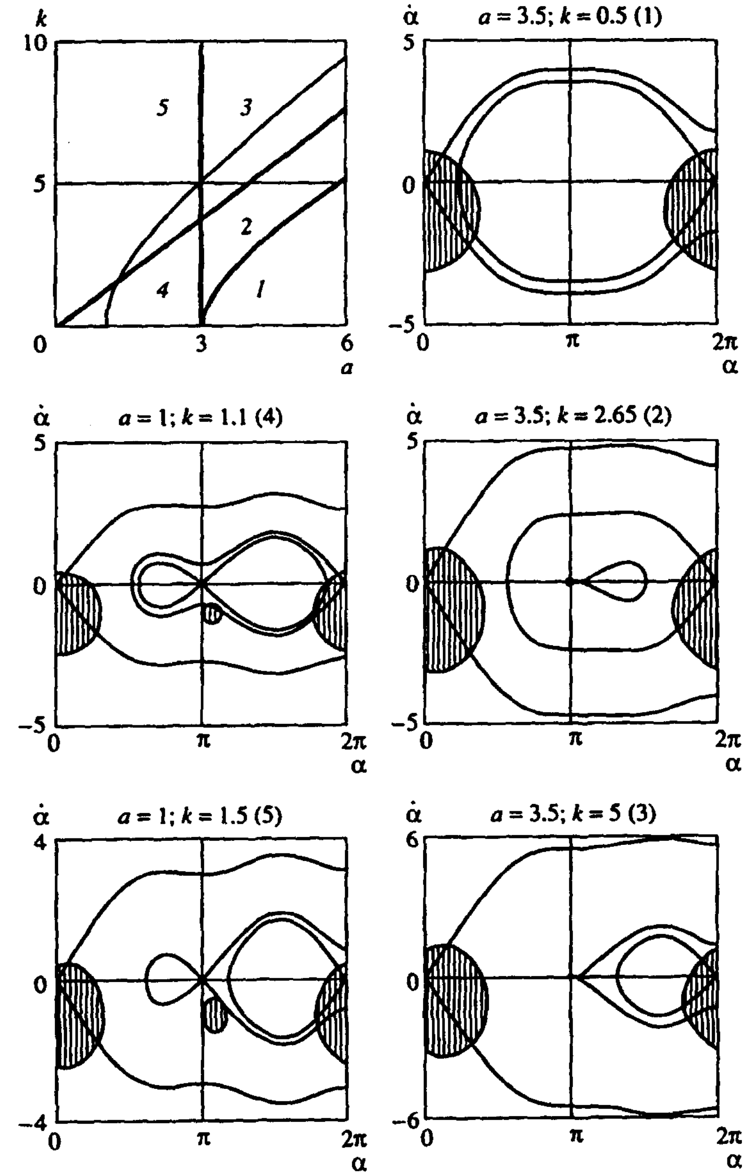

Fig. 2

We recall that if a phase trajectory happens to enter the zone of leaving the constraint, this means that further motion of the couple is not described by Eq. (4.1) of coupled motion.

In cases 2 and 3 there are two separatrices on the phase cylinder. One emerges from the saddle $\alpha = 0$ and the other from the saddle $\alpha = \pi$ (if $a < 3$) or from the saddle $\alpha = \alpha_-$ (if $a > 3$).

The phase portrait depends, qualitatively speaking, on the form of the second separatrix. Depending on the values of the parameters a and k, it may be bounded (having the form of a "recumbent figure of eight"), or, like the first separatrix, it may have unbounded "whiskers". These two subcases are separated by the singular case of a single separatrix passing through all saddle points. This happens if the values of the parameters a and k lie on the curve

$$k = F(a) = \begin{cases} 4a/\pi, & a < 3 \\ f(a), & a > 3 \end{cases}$$

where $f(a)$ is defined parametrically by the formulae

$$k(\beta) = Z(1-\cos\beta)^2, \quad a(\beta) = Z(\sin\beta - b\cos\beta)$$
$$Z = \frac{3}{\beta - \sin\beta + \sin\beta(\cos\beta - 1)} \tag{4.9}$$

The parameter β varies in the range from $\beta = \pi$ (when $k = 12/\pi$, $a = 3$) to $\beta = \beta_0$, where $\beta_0 \approx 123°$ is the solution of the equation obtained by equating the expression for Z to zero. At $\beta = \beta_0$ we have $k \to \infty$, $a \to \infty$. Note that when $a \geqslant 3$ formulae (4.9) lead, with good accuracy, to the formula $k = 4a/\pi$ over a fairly large range of values of a and k.

The (a, k) parameter plane is divided by the curve $k = \sqrt{a^2-9}$, $a = 3$, $k = F(a)$ into five domains (the upper left part of Fig. 2) of qualitatively distinct phase portraits. Examples of these portraits are shown in Fig. 2, numbered from 1 to 5 (in the neighbourhoods of the centres of Figs 2(2) and 2(3) are small loops of separatrices). The zones of leaving the constraints, that is, corresponding to the inequality opposite to (4.2), are shown hatched.

There may be two zones of leaving the constraint (in the neighbourhood of $\alpha = 0$ and $\alpha = \pi$) or one (in the neighbourhood of $\alpha = 0$ only). In the (a, k) parameter plane the domain of existence of one zone of leaving the constraint is separated from the existence domain of two zones by a curve defined parametrically by the equations

$$a = -\frac{2-\cos^2\alpha}{\cos^3\alpha}, \quad k = -\frac{\sin\alpha(2+3\cos^2\alpha)}{\cos^3\alpha} \tag{4.10}$$

The quantities a and k are defined (and positive) as the parameter α decreases from $\alpha = \pi$ to $\alpha = \pi/2$; when $\alpha = \pi$ we have $a = 1$, $k = 0$; as $\alpha \to \pi/2$ we have $a \to \infty$, $k \to \infty$. The curve (4.10) is represented in Fig. 2 by a thin line.

The above investigation has enabled us, in particular, to observe the following outcomes of the aerogradient effect (Fig. 2).

1. An orbital tethered system may spin up to a fairly high angular velocity (unlimited when there is no dissipation).

2. A stable "lower" position ($\alpha \sim \pi/2$) of a probe is possible, but the stability domain is reduced by the aerogradient effect.

3. The stability domain of the "upper" position ($\alpha \sim 3\pi/2$) of a probe is enlarged by the aerogradient effect.

5. REGULARITY AND CHAOS IN THE MOTION OF A DUMB-BELL SATELLITE IN AN ELLIPTIC ORBIT

The principal qualitative effect in the rotational motion of a satellite in an elliptic orbit is the possibility of chaotization of the motion [15–18].†

The atmosphere particularly strongly affects the onset of chaotization because of the exponential variation of its density along an elliptic orbit. Even relatively small eccentricities imply strong chaotization.

In Fig. 3 we show phase portraits of the solutions for Eq. (1.2) in the α, $\dot{\alpha}$plane, for the case in which the gravitational torques and forces of aerodynamic pressure are effective (that is, it is assumed in (1.2) that $a \neq 0$, $e \neq 0$, but $b = k = 0$). The phase portraits were constructed by numerical implementation of the method of point mappings over one revolution in the orbit. Corresponding to chaotic motion are the domains in these figures continuously filled with points; regular (conditionally periodic) oscillations and rotations of the satellite are represented by solid curves, and periodic motions by centres of islands of regular motions. In view of the symmetry of the phase portraits about the ordinate axis, all but two are represented by their halves corresponding to the interval $\alpha[-\pi, 0]$.

The upper part shows phase portraits for an almost circular orbit ($e = 10^{-3}$) for different values of the aerodynamic parameter a. For a small value of the parameter ($a = 0.05$), the thin chaotic layers seen in Fig. 3 form in the neighbourhood of each of the two separatrices of the phase portrait for a circular orbit. The centres of the islands correspond to stable periodic oscillations about the radial

†See also BELETSKII, V. V.. Regular and chaotic motions in the problem of the attitude control of an artificial satellite. Preprint No. 53. M. V. Keldysh, Inst. Prikl. Mat. Akad. Nauk SSSR, Moscow, 1990.

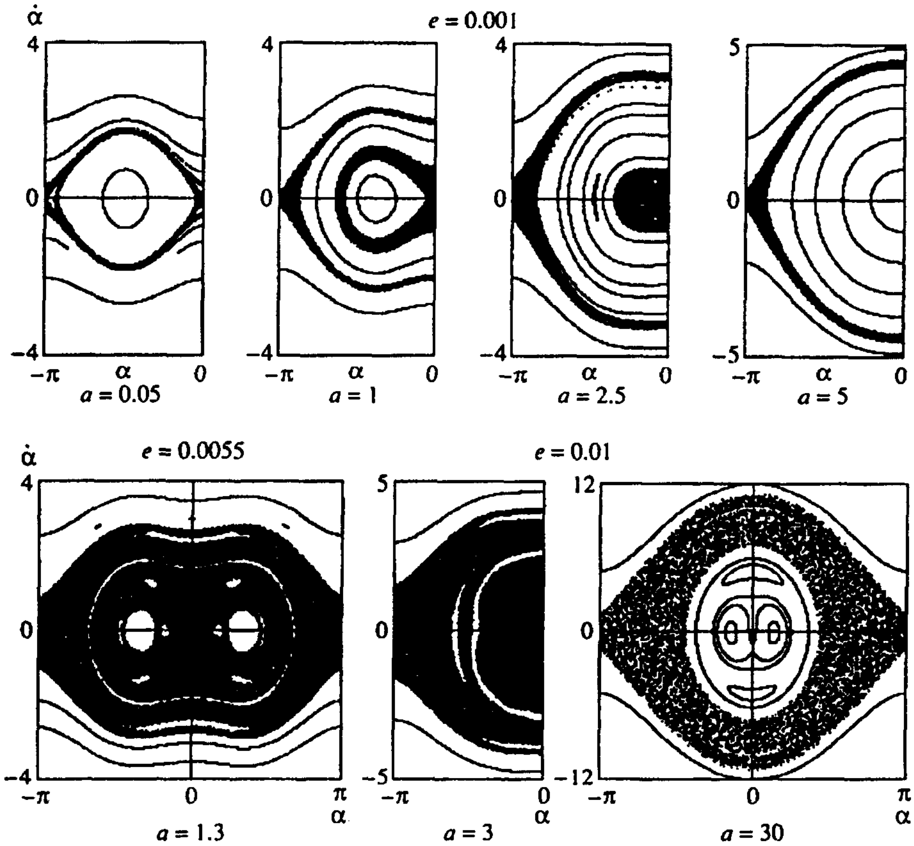

Fig. 3

direction (toward the Earth or away from the Earth). The small arcs with maxima at $\alpha = -1$ are very thin islets corresponding to oscillations about a fixed direction in space, collinear with the minor axis of the ellipse. When $a = 1$ the chaotic layers generated by each of the two separatrices are clearly visible. When $a = 2.5$ the inner chaotic layer becomes a chaotic sea (in which one observes an archipelago of weakly stable oscillations about $\alpha = 0$). A relatively high value for the aerodynamic parameter, $a = 5$, gives a domain of aerodynamic stabilization.

Thus, a very low ellipticity of the orbit ($e \sim 10^{-3}$) gives a relatively satisfactory picture: for example, the small oscillations about the radial direction remain small and regular at relatively low values of the aerodynamic parameter, the regularity zone remains fairly large, and so on.

This "satisfactory" picture rapidly disappears as the eccentricity is increased ($e = 0.0055$, $a = 1.3$). The two chaotic layers have almost coalesced into a uniform sea, forming only a thin "atoll" of regularity between the two chaotic layers; the island of regularity within the sea has become substantially smaller. When $e = 0.01$ the figure illustrates the evolution of the phase portrait as a increases ($a = 1$ and $a = 30$). It is interesting that very large values of a regularize the phase pattern. However, at moderate values $a \sim 1$–3 the motion is strongly chaotic.

Generally speaking, chaotization increases as e increases, and when $a \sim 1$ a continuous sea of chaotic trajectories forms even when $e \sim 0.1$.

We wish to thank V. L. Vorontsova and A. A. Savchenko for verifying some of the results presented in this paper.

This research was supported financially by the Russian Foundation for Basic Research (98-01-00940) and the International Association for Promoting Cooperation with Scientists from the New Independent States of the Former Soviet Union (INTAS 94-644).

REFERENCES

1. BELETSKII, V. V., and LEVIN, Ye. M., *Dynamics of Space Tethered Systems*. Nauka, Moscow, 1990.
2. BELETSKII, V. V., and YASHIN, A. M., *The Effect of Aerodynamic Forces on the Rotational Motion of Artificial Satellites*. Naukova Dumka, Kiev, 1984.
3. BELETSKII, V. V., *Essays on the Motion of Cosmic Bodies*. Nauka, Moscow, 1977.
4. BELETSKII, V. V., *The Motion of an Artificial Satellite about the Centre of Mass*. Nauka, Moscow, 1965.
5. SARYCHEV V. A., Problems of Attitude Control of Artificial Satellites (*Advances in Science and Technology. Space Research*, No. 11). VINITI, Moscow, 1978.
6. BELETSKII, V. V., and PANKOVA, D. V., A two-body system in orbit as a dynamic billiard. *Regulyarnaya i Khaoticheskaya Dinamika*, 1996, **1**, 1, 87–103.
7. BELETSKII, V. V., and NOVIKOVA, Ye. T., The relative motion of a two-body system in orbit. *Kosmich. Issled.*, 1969, **7**, 3, 377–384.
8. BELETSKII, V. V., The relative motion of a two-body system in orbit. II. *Kosmich. Issled.*, 1969, **7**, 6, 827–840.
9. DOKUCHAYEV, L. V. and YEFIMENKO, G. G., The influence of the atmosphere on the relative motion of a two-body system in orbit. *Kosmich. Issled.*, 1972, **10**, 1, 57–65.
10. BELETSKII, V. V. and VORONTSOVA, V. L., The influence of the density gradient of the atmosphere on the spin and attitude of a dumb-bell satellite. *Vestnik Mosk. Gos. Univ., Ser. Mat. Mekh.*, 2000, 5, 35–39.
11. BELETSKII, V. V., The libration of a satellite. In *Artificial Earth Satellites*. Izd. Akad. Nauk SSSR, Moscow, No. 3 1959, 13–31.
12. SINGKH, R. B., The motion of connected bodies in an elliptic orbit. *Vestnik Mosk. Gos. Univ., Ser. Mat. Mekh.*, 1973, 3, 82–86.
13. VORONTSOVA, V. L., Investigation of coupled motion and connectedness condition ignoring aerodynamic dissipation. *Vestnik Stavropol. Univ.*, 1999, 18, 49–56.
14. BARBASHIN, Ye. A. and TABUYEVA, V. A., *Dynamical Systems with Cylindrical Phase Space*. Nauka, Moscow, 1969.
15. WISDOM, J., PEALE, S. J., and MIGNARD, F., The chaotic rotation of *Hyperion. Icarus*, 1984, **58**, 2, 137–152.
16. WISDOM, J., Rotational dynamics of irregularly shaped natural satellites. *Astron. J.*, 1987, **94**, 5, 1350–1360.
17. BELETSKY, V. V., *Reguläre und chaotische Bewegung starrer Körper*. Teubner, Stuttgart, 1995.
18. BELETSKII, V. V., PIVOVAROV, M. L. and STAROSTIN, E. L., Regular and chaotic motions in applied dynamics of a rigid body. *Chaos*, 1996, **6**, 2, 155–166.

Translated by D.L.